Stress Concentration Factor Formulas Useful for Any Dimensions of Shoulder Fillet in a Flat Test Specimen Under Tension and Bending

Article in Journal of Testing and Evaluation · May 2004
DOI: 10.1520/JTE11799

Nao-Aki Noda
Kyushu Institute of Technology

Journal of Testing and Evaluation, May 2004, Vol. 32, No. 3
Paper ID JTE11799
Available online at: www.astm.org

N. A. Noda[1] *and Y. Takase*[1]

Stress Concentration Factor Formulas Useful for Any Dimensions of Shoulder Fillet in a Flat Test Specimen Under Tension and Bending

ABSTRACT: In this work stress concentration factors K_t for a flat bar with fillets are considered on the basis of exact solutions now available for special cases and accurate numerical results. Then, a convenient K_t formula useful for any dimensions of the fillet is proposed. The conclusions can be summarized as follows.

1. For the limiting cases of deep (d) and shallow (s) fillets, the body force method is used to calculate the K_t values. Then, the formulas are obtained as K_{td} and K_{ts}.
2. On the one hand, upon comparison of K_t and K_{td}, it is found that K_t is nearly equal to K_{td} if the fillet is deep or blunt.
3. On the other hand, if the fillet is sharp or shallow, K_t is controlled mainly by K_{ts} and the fillet depth.
4. The fillet shape is classified into several groups according to the fillet radius and fillet depth. Then, the least-squares method is applied for calculation of K_t/K_{td} and K_t/K_{ts}.
5. Finally, convenient formulas are proposed that are useful for any dimensions of fillet in a flat bar. The formulas give SCFs with less than about 1 % error in most cases for any dimensions of fillet under tension and bending.

KEYWORDS: stress concentration factor, shoulder fillet, flat bar, test specimen, tension, bending, body force method

Nomenclature

- a Length of minimum section
- D Length of maximum section
- t Depth of fillet
- K_t Stress concentration factor (SCF) based on minimum section shown in Fig. 1*a*, *b*, *c* ($K_t = \sigma_{max}/\sigma_n$ in tension: $\sigma_n = P/2a$, P = tensile load; in bending: $\sigma_n = 6M/(2a)^2$, M = bending moment)
- K_{td} Exact solution for SCF of deep fillet when $2t/D \to 1.0$
- K_{tE} SCF of an elliptical hole in an infinite plate under uniform tension when lower rim of hole is also subjected to traction $\sigma (=1 + \sqrt{t/\rho})$
- K_{tH} SCF of hyperboloidal notch
- K_{ts} Exact solution for SCF of shallow fillet when $2t/D \to 0$
- x $x = a/\rho$, when $a/\rho \leq 1.0$; $x = 2 - \rho/a$, when $\rho/a \leq 1.0$
- η $= \sqrt{\rho/t}$
- λ Relative fillet depth $= 2t/D$
- ν Poisson's ratio (=0.3)
- ξ $= \sqrt{t/\rho}$
- ρ Root radius of fillet

Manuscript received 18 Nov. 2002; accepted for publication 14 Jan. 2004; published May 2004.

[1] Department of Mechanical Engineering, Kyushu Institute of Technology, Kitakyushu 804-8550, Japan. E-mail: noda@kyutech.ac.jp

Introduction

Several formulas have been proposed on the basis of the stress concentration analysis of notches [1–3]. Stress concentration analysis of shoulder fillets is, however, one of the most important problems in the design of high-performance structures where both light weight and high strength are desirable. Also, it is important for designing test specimens for tension test and fatigue experiments. Usually, analysis of fillets is more difficult than analysis of notches because the position where the maximum stress appears changes depending on the dimensions of the shoulder fillets. In our previous studies [4,5], accurate stress concentration factors were given using analysis by the body force method. The accuracy is estimated to be within 1 % in the range of $0.03 \leq 2\rho/D \leq 1.0$. Then, the results have shown that Peterson's stress concentration factors [6] have a nonconservative error of about 10 % for a wide geometrical range of fillets. However, accurate SCFs were not given in the form of formulas suitable for engineering applications.

Instead, for notched bars, Neuber proposed a simple ingenious approximate formula K_{tN} useful for a wide range of notch shapes [7]. In the preceding paper, therefore, for the problem of shoulder fillets, similar equations were also proposed as an extension of Neuber's formula. Then, more accurate formulas were proposed [8] using the results of the body force method and correcting Neuber's values. However, the formulas are useful for only a certain range of fillet dimensions because of the restrictions of the body force method application [4,5]. Generally, there is no numerical method that is effective for analyzing any shape of notch with less than 1 % error. In other words, it is difficult to obtain accurate stress concentration factors when the notch root radius is extremely large or extremely small. Therefore, in the preceding study [9], K_t values

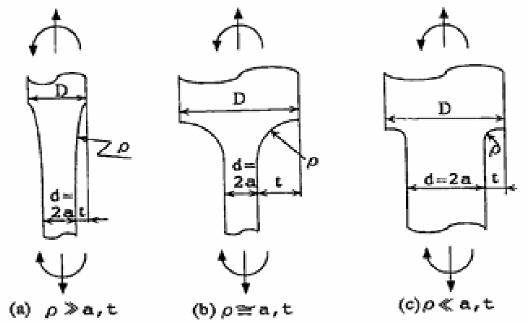


FIG. 1—*Flat specimens with a circular-arc or a V-shaped fillet.* (a) *Extremely blunt fillet.* (b) *Ordinary fillet.* (c) *Extremely sharp fillet.*

for any dimensions of the notch were given using the exact solutions for special cases and accurate numerical results.

In a similar way, in this study, first, K_t of a flat bar with fillets will be considered for limiting cases of deep and shallow fillets, K_{td} and K_{ts}. Second, it will be shown that if the fillet is sharp or shallow, K_t is controlled mainly by K_{ts} with the depth of the shoulder fillet. Third, upon comparison of K_t and K_{td} it will be found that K_t is nearly equal to K_{td} if the fillet is deep or blunt. Fourth, the fillet shape will be classified into several groups according to the fillet radius and fillet depth. Finally, a set of convenient formulas useful for any shape of fillet will be proposed by applying the least-squares method for each group. The set of formulas yields SCFs with less than 1 % error in most cases. Note that the previous formulas [5,8] can be applied only within the range of $0.03 \leq 2\rho/D \leq 1.0$. However, the present formulas can be applied to any dimensions of fillet.

Stress Concentration Factors of Sharp or Shallow Fillets

Consider the stress concentration factor K_t of a fillet as shown in Fig. 1*c*. In this case the SCF of the sharp fillet is controlled mainly by the SCF of a fillet in a semi-infinite plate K_{ts} having the same shape ratio t/ρ. The ratio K_t/K_{ts} is shown in Fig. 2 where the line for $\rho/a \to 0$ is obtained from results when ρ/a is small; for example, $\rho/a = 0.1, 0.05$, confirming that the two results are in agreement to the third digit. As shown in Fig. 2, the value of K_t/K_{ts} is in a narrow range in tension compared with that in bending. That means the K_{ts} approximation is better in tension than in bending. In Fig. 2, we can see that:

1. In sharp fillets ($\rho/a \leq 0.1$), the value of K_t/K_{ts} is determined by $2t/D$ alone. The value of K_t can be obtained from K_{ts} in the following range depending on the loading conditions: (*a*) tension: $\rho/a \leqq 0.1, 2t/D \leqq 0.9$; (*b*) bending: $\rho/a \leqq 0.1, 2t/D \leqq 0.9$.
2. In shallow fillets ($2t/D \leqq 0.02$), the value of K_t/K_{ts} is controlled by $2t/D$ for a wide range of a/ρ values. (*a*) tension: The value of K_t/K_{ts} is determined by $2t/D$ alone when $a/\rho \geqq 0.01$. (*b*) bending: The value of K_t/K_{ts} is within a small range, when $a/\rho \geqq 0.01$.
3. The stress concentration factor of blunt and shallow fillets ($2t/D \leqq 0.02$) will be considered in the next section but can be estimated as follows: (*a*) tension: $a/\rho \leqq 0.01$, $K_t = (1.000 \sim 1.005)$; (*b*) bending: $a/\rho \leqq 0.01$, $K_t = (1.000 \sim 1.008)$.

Stress Concentration Factors of Blunt and Deep Fillets

Here, the SCF is considered for blunt and deep fillets. The SCF of a deep hyperboloidal notch K_{tH} [7] can be used as a good approximate formula. The deep fillet ($2t/D \geqq 0.7$) can be estimated as an infinitely deep fillet as shown in Fig. 3. Table 1 gives ratios K_t/K_{tH} when $2t/D = 0.7, 0.8, 0.9$.

With increasing fillet depth, K_t/K_{tH} approaches limiting values. That is, as $2t/D \to 1.0$, $K_t/K_{tH} \to (0.910 \sim 1.000)$ in tension and $K_t/K_{tH} \to (0.842 \sim 1.000)$ in bending. The ratio K_t/K_{tH} does not approach unity because of the difference of the shape—shoulder fillet and hyperboloid. The Table 1 the limiting values of K_{td}/K_{tH} are obtained using the convergence of K_t/K_{tH} when $2t/D \to 1.0$. The approximate formulas for a deep fillet can be expressed as a function of the parameter x of Table 1 by applying the least-squares method to the results given in Table 1. The K_t/K_{tH} value when $a/\rho \to \infty$ is obtained using extrapolation with corresponding good accuracy; for example, an extrapolated value from $a/\rho = 3.333$ to 5, and another extrapolated value from $a/\rho = 5$ to 10 coincide with each other to the third digit in some cases.

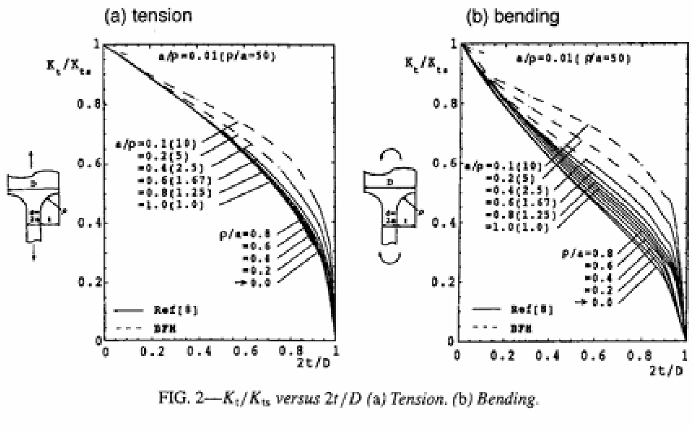


FIG. 2—K_t/K_{ts} *versus* $2t/D$ (a) *Tension.* (b) *Bending.*

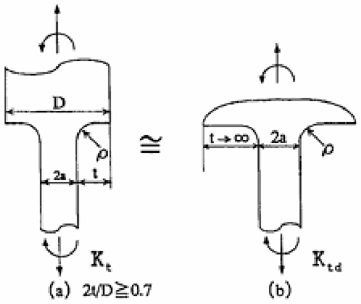


FIG. 3—$K_t \cong K_{td}$ *when* $2t/D \geqq 0.7$.

TABLE 1—K_t/K_{td1} *when* $2t/D \rightarrow 1.0$. (a) *Tension* (b) *Bending.*

x	a/ρ	ρ/a	$2t/D = 0.7$	$2t/D = 0.8$	$2t/D = 0.9$	$2t/D \rightarrow 1.0$
(*a*) Tension						
0.000	0.000	∞				1.000
0.100	0.100	10.00	0.991	0.991	0.991	0.991
0.200	0.200	5.000	0.984	0.983	0.983	0.982
0.300	0.300	3.333	0.980	0.978	0.976	0.974
0.400	0.400	2.500	0.981	0.977	0.973	0.971
0.500	0.500	2.000	0.982	0.978	0.973	0.968
0.600	0.600	1.667	0.984	0.979	0.974	0.969
0.700	0.700	1.429	0.984	0.981	0.975	0.969
0.800	0.800	1.250	0.985	0.982	0.977	0.973
0.900	0.900	1.111	0.984	0.983	0.979	0.973
1.000	1.000	1.000	0.984	0.984	0.980	0.976
1.000	1.000	1.000	0.984	0.984	0.980	0.976
1.100	1.111	0.900	0.983	0.984	0.982	0.982
1.200	1.250	0.800	0.981	0.983	0.983	0.983
1.300	1.429	0.700	0.978	0.982	0.983	0.984
1.400	1.667	0.600	0.973	0.980	0.983	0.986
1.500	2.000	0.500	0.966	0.976	0.981	0.986
1.600	2.500	0.400	0.955	0.968	0.976	0.984
1.700	3.333	0.300	0.939	0.955	0.968	0.979
1.800	5.000	0.200	0.911	0.933	0.951	(0.969)
1.850	6.667	0.150	0.892	0.915		(0.961)
1.900	10.00	0.100	0.863	0.890		(0.942)
2.000	∞	0.000				(0.910)
(*b*) Bending						
0.000	0.000	∞				1.000
0.100	0.100	10.00	0.993	0.993	0.993	0.993
0.200	0.200	5.000	0.983	0.983	0.985	0.987
0.300	0.300	3.333	0.982	0.982	0.983	0.984
0.400	0.400	2.500	0.977	0.977	0.980	0.983
0.500	0.500	2.000	0.973	0.973	0.976	0.979
0.600	0.600	1.667	0.968	0.968	0.972	0.974
0.700	0.700	1.429	0.967	0.967	0.967	0.967
0.800	0.800	1.250	0.961	0.961	0.961	0.961
0.900	0.900	1.111	0.958	0.958	0.958	0.958
1.000	1.000	1.000	0.957	0.957	0.957	0.957
1.000	1.000	1.000	0.957	0.957	0.957	0.957
1.100	1.111	0.900	0.952	0.952	0.951	0.951
1.200	1.250	0.800	0.948	0.948	0.947	0.947
1.300	1.429	0.700	0.945	0.944	0.943	0.942
1.400	1.667	0.600	0.941	0.938	0.938	0.938
1.500	2.000	0.500	0.933	0.932	0.932	0.932
1.600	2.500	0.400	0.925	0.924	0.924	0.924
1.700	3.333	0.300	0.914	0.913	0.913	0.913
1.800	5.000	0.200	0.897	0.896	0.898	(0.898)
1.850	6.667	0.150	0.884	0.886		(0.888)
1.900	10.00	0.100	0.867	0.869		(0.870)
2.000	∞	0.000				(0.842)

Using the K_{td} formula, the ratio K_t/K_{td} is shown in Fig. 4. The K_{td} approximation is better in bending than in tension as shown in Fig. 4. From Fig. 4, the K_{td} approximation is useful in the following range.

(*a*) Tension:
1. $2t/D \geqq 0.9$, $10 \leqq a/\rho \leqq \infty$; $0.971 \leqq K_t/K_{td} \leqq 1.000$
2. $0.7 \leqq 2t/D \leqq 1.0$, $0 \leqq a/\rho \leqq 10$; $0.919 \leqq K_t/K_{td} \leqq 1.022$
3. $0.02 \leqq 2t/D \leqq 0.7$, $0 \leqq a/\rho \leqq 0.3$; $0.930 \leqq K_t/K_{td} \leqq 1.016$

(*b*) Bending:
1. $2t/D \geqq 0.9$, $10 \leqq a/\rho \leqq \infty$; $0.996 \leqq K_t/K_{td} \leqq 1.000$
2. $0.4 \leqq 2t/D \leqq 1.0$, $0 \leqq a/\rho \leqq 10$; $0.961 \leqq K_t/K_{td} \leqq 1.017$
3. $0.02 \leqq 2t/D \leqq 0.4$, $0 \leqq a/\rho \leqq 0.6$; $0.936 \leqq K_t/K_{td} \leqq 1.000$

Stress Concentration Factors of Other Fillets

Here, the stress concentration factor K_t is considered for the remaining region: for tension $0.1 \leqq \rho/a \leqq 3.3$, $0.02 \leqq 2t/D \leqq 0.7$; for bending $0.1 \leqq \rho/a \leqq 1.67$, $0.02 \leqq 2t/D \leqq 0.4$). In this region, as shown in Fig. 2 the value of K_t/K_{ts} is determined by $2t/D$ and exists in a narrow range. Figure 5 shows $(K_t/K_{ts})/[K_t/K_{ts}]_{\rho/a=1.25}$ in tension and $(K_t/K_{ts})/[K_t/K_{ts}]_{\rho/a=0.6}$ in bending. They are useful in the following range: (*a*) tension: $0.1 \leqq \rho/a \leqq 3.3$, $0.02 \leqq 2t/D \leqq 0.7$; $0.98 \leqq (K_t/K_{ts})/([K_t/K_{ts}]_{\rho/a=1.25}) \leqq 1.07$. (*b*) bending: $0.1 \leqq \rho/a \leqq 1.67$, $0.02 \leqq 2t/D \leqq 0.4$; $0.95 \leqq (K_t/K_{ts})/([K_t/K_{ts}]_{\rho/a=0.6}) \leqq 1.05$.

A Set of SCF Formulas Useful for any Dimensions of Fillet

From the above discussion, any shape of fillet can be classified into one of the groups shown in Fig. 6. Then, the least-squares method is applied to each region shown in Fig. 6. Finally, a set of accurate formulas for the whole range of fillet shapes is obtained. The results are as follows:

SCF of Fillet in a Semi-Infinite Plate K_{ts}

The limiting SCF of a shallow fillet $K_t \rightarrow K_{ts}$ in Fig. 1 when $a, D \rightarrow \infty$. The K_{ts} formula can be expressed in Eq 1 which was proposed in Refs 5,8:

$$K_{ts}/K_{tE} = 1.000 + 0.159\xi - 0.127\xi^2 + 0.050\xi^3 \quad (0 \leqq \xi \leqq 1) \quad (1a)$$

$$K_{ts}/K_{tE} = 1.106 + 0.016\eta - 0.059\eta^2 + 0.019\eta^3 \quad (0 \leqq \eta \leqq 1) \quad (1b)$$

$$\xi = \sqrt{t/\rho},\ \eta = \sqrt{\rho/t},\ K_{tE} = 1 + \sqrt{t/\rho} \quad (1c)$$

SCF of Sharp or Shallow Fillet K_t

Tension—Region 1 in Fig. 6*a*: $\rho/a \leqq 0.1$ and $2t/D \leqq 0.9$, or $a/\rho \geqq 0.01$ and $2t/D \leqq 0.02$. The SCF for a sharp or a shallow fillet in tension can be expressed by Eq 2. The value of Eq 2 is shown in Fig. 7*a*:

$$K_t/K_{ts} = 0.99902 - 0.028181\lambda - 11.872\lambda^2 + 102.26\lambda^3 - 435.32\lambda^4 + 1007.3\lambda^5 - 1293.8\lambda^6 + 866.03\lambda^7 - 235.60\lambda^8 \quad (2)$$

Bending—Region 1 in Fig. 6*b*: $\rho/a \leqq 0.1$ and $2t/D \leqq 0.9$, or $a/\rho \geqq 0.01$ and $2t/D \leqq 0.02$. The SCF for a sharp fillet in bending

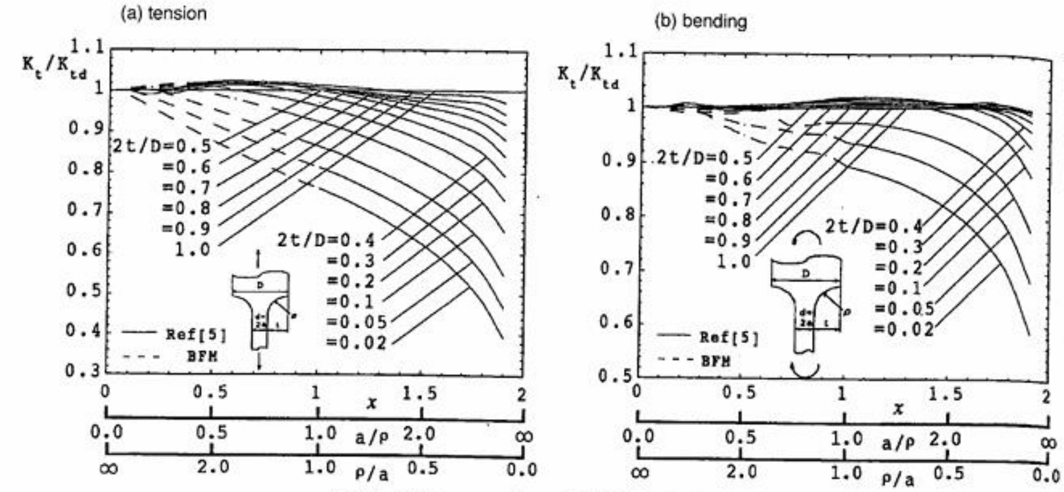


FIG. 4—K_t/K_{td} *versus* a/ρ *or* ρ/a (a) *Tension.* (b) *Bending.*

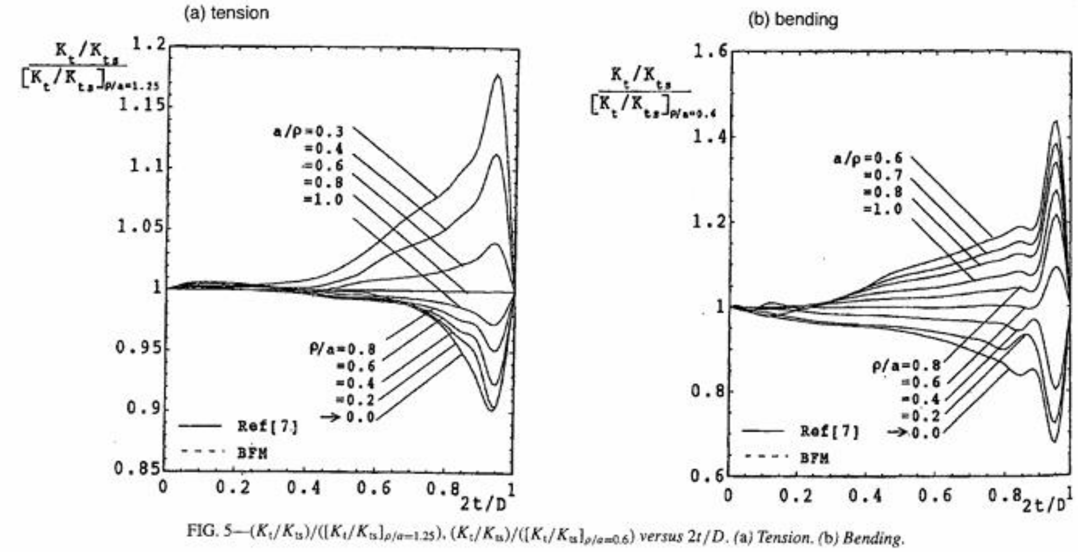


FIG. 5—$(K_t/K_{ts})/([K_t/K_{ts}]_{\rho/a=1.25})$, $(K_t/K_{ts})/([K_t/K_{ts}]_{\rho/a=0.6})$ *versus* $2t/D$. (a) *Tension.* (b) *Bending.*

can be expressed by Eq 3. The value of Eq 3 is shown in Fig. 7*b*.

$$K_t/K_{ts} = 1.0197 - 2.0185\lambda + 10.241\lambda^2 - 52.694\lambda^3 + 159.98\lambda^4 - 286.24\lambda^5 + 297.61\lambda^6 - 166.33\lambda^7 + 38.533\lambda^8 \quad (3)$$

SCF of Blunt and Shallow Fillet (K_t)

Tension—Region 6 in Fig. 6*a*: $a/\rho \leqq 0.01$ and $2t/D \leqq 0.02$

$$K_t \cong 1.002 \quad (4)$$

Bending—Region 6 in Fig. 6*b*: $a/\rho \leqq 0.01$ and $2t/D \leqq 0.02$

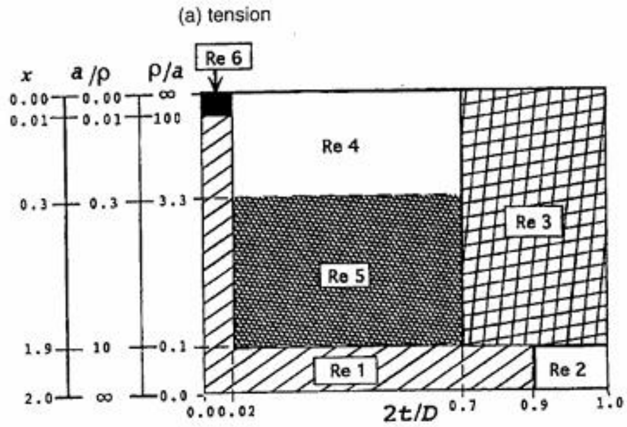
$$K_t \cong 1.004 \quad (5)$$

SCF of a Deep Fillet K_{td} *as Shown in Fig. 5*b

The limiting SCF of a deep fillet $K_t \rightarrow K_{td}$ when $t \rightarrow \infty$. The K_{td} formula can be expressed in Eqs 6 and 7. Figure 8 shows the values of Eqs 6 and 7 from the results of the body force method:

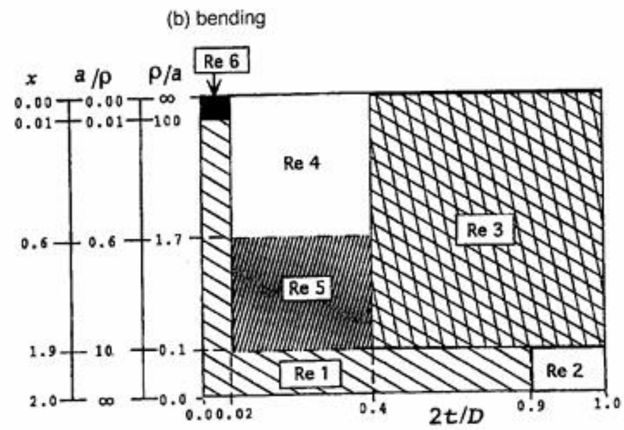
Tension—

$$K_{td}/K_{tH} = 0.99841 - 0.081553x + 0.010983x^2 + 0.091256x^3 - 0.043397x^4 \quad (6a)$$



Region 1 *Sharp or shallow fillet*
Region 2 *Sharp and deep fillet*
Region 3 *Deep fillet*
Region 4 *Blunt fillet*
Region 5 *Ordinary fillet*
Region 6 *Blunt and shallow fillet*

FIG. 6*a*—*Classification of fillet shape in a flat bar under tension.*



Region 1 *Sharp or shallow fillet*
Region 2 *Sharp and deep fillet*
Region 3 *Deep fillet*
Region 4 *Blunt fillet*
Region 5 *Ordinary fillet*
Region 6 *Shallow fillet*
Region 7 *Blunt and shallow fillet*

FIG. 6*b*—*Classification of fillet shape in flat bar under bending.*

where

$$K_{tH} = \frac{2(a/\rho + 1)\sqrt{a/\rho}}{(a/\rho + 1)\tan^{-1}\sqrt{a/\rho} + \sqrt{a/\rho}} \tag{6b}$$

Bending—

$$K_{td}/K_{tH} = 0.99618 - 0.0013640x - 0.13691x^2 + 0.14372x^3 - 0.046969x^4 \tag{7a}$$

$$K_{tH} = \frac{4a/\rho \times \sqrt{a/\rho}}{3\{\sqrt{a/\rho} + (a/\rho - 1)\tan^{-1}\sqrt{a/\rho}\}} \tag{7b}$$

SCF of a Sharp and a Deep Fillet, K_t

Tension—Region 2 in Fig. 6*a*: $\rho/a \leqq 0.1$ and $2t/D \geqq 0.9$

$$K_t/K_{td} \cong \text{Eqs } 3 \times 0.986 \tag{8}$$

Bending—Region 2 in Fig. 6*b*: $\rho/a \leqq 0.1$ and $2t/D \geqq 0.9$

$$K_t/K_{td} \cong \text{Eqs } 3 \times 0.998 \tag{9}$$

SCF of Deep Fillet K_t

Tension—Region 3 in Fig. 6*a*: $2t/D \geqq 0.7$ and $a/\rho \leqq 10\ (0 \leqq x \leqq 1.9)$. The SCF of a deep fillet can be expressed by Eq 10. Figure 9*a* shows the value of Eq 10:

$$\begin{aligned} K_t/K_{td} = {} & -0.31350 + 4.8845\lambda - 6.0210\lambda^2 + 2.4500\lambda^3 \\ & + (15.696 - 57.427\lambda + 69.681\lambda^2 - 27.950\lambda^3)x \\ & + (-16.126 + 58.883\lambda - 71.191\lambda^2 + 28.433\lambda^3)x^2 \\ & + (4.3528 - 15.969\lambda + 19.295\lambda^2 - 7.6783\lambda^3)x^3 \end{aligned} \tag{10}$$

Bending—Region 3 in Fig. 6*b*: $2t/D \geqq 0.4$ and $a/\rho \leqq 10\ (0 \leqq x \leqq 1.9)$. The SCF of a deep fillet can be expressed by Eq 11. Figure 9*b* shows the values of Eq 11:

$$\begin{aligned} K_t/K_{td} = {} & 0.04669493 + 9.718178\lambda - 40.25930\lambda^2 + 86.85160\lambda^3 \\ & - 102.9267\lambda^4 + 63.54189\lambda^5 - 15.97227\lambda^6 \\ & + (22.38870 - 219.7406\lambda + 877.2032\lambda^2 - 1823.727\lambda^3 \\ & + 2083.285\lambda^4 - 1240.427\lambda^5 + 301.0174\lambda^6)x \\ & + (-64.65332 + 627.4616\lambda - 2479.609\lambda^2 \\ & + 5110.695\lambda^3 - 5798.301\lambda^4 + 3435.843\lambda^5 \\ & - 831.4363\lambda^6)x^2 + (55.86065 - 538.9289\lambda \\ & + 2121.100\lambda^2 - 4358.482\lambda^3 + 4933.191\lambda^4 \\ & - 2917.695\lambda^5 + 704.9552\lambda^6)x^3 + (-14.93828 \\ & + 143.4999\lambda - 563.4847\lambda^2 + 1156.195\lambda^3 \\ & - 1307.170\lambda^4 + 772.2462\lambda^5 - 186.3483\lambda^6)x^4 \end{aligned} \tag{11}$$

SCF of Blunt Fillet K_t

Tension—Region 4 in Fig. 6*a*: $a/p \leqq 0.3\ (0 \leqq x \leqq 0.3)$ and $0.02 \leqq 2t/D \leqq 0.4$. The SCF of a blunt fillet in tension can be expressed by Eq 12. Figure 10*a* shows the value of Eq 12:

$$\begin{aligned} K_t/K_{td} = {} & 1.0026 - 0.038276\lambda + 0.17613\lambda^2 - 0.25762\lambda^3 \\ & + 0.032396\lambda^4 + 0.11455\lambda^5 \\ & + (-0.052912 + 4.0544\lambda - 35.308\lambda^2 + 111.63\lambda^3 \\ & - 151.03\lambda^4 + 74.331\lambda^5)x \\ & + (-2.1823 + 20.748\lambda - 28.428\lambda^2 - 79.661\lambda^3 \\ & + 209.11\lambda^4 - 127.41\lambda^5)x^2 \\ & + (4.3447 - 64.393\lambda + 242.67\lambda^2 - 380.49\lambda^3 \\ & + 272.07\lambda^4 - 74.580\lambda^5)x^3 \end{aligned} \tag{12}$$

Bending—Region 4 in Fig. 6*b*: $a/\rho \leqq 0.6\ (0 \leqq x \leqq 0.6)$ and $0.02 \leqq 2t/D \leqq 0.4$. The SCF of a blunt fillet in bending can be expressed by Eq 13. Figure 10*b* shows the values of Eq 13:

$$\begin{aligned} K_t/K_{td} = {} & 0.99733 + 0.075203\lambda - 0.65862\lambda^2 \\ & + 2.1384\lambda^3 - 2.2928\lambda^4 + (0.21790 - 5.8953\lambda \\ & + 44.163\lambda^2 - 131.98\lambda^3 + 136.68\lambda^4)x + (-1.0526 \\ & + 21.568\lambda - 142.21\lambda^2 + 392.64\lambda^3 - 388.88\lambda^4)x^2 \\ & + (0.69399 - 10.748\lambda + 54.029\lambda^2 + 118.25\lambda^3 \\ & + 101.18\lambda^4)x^3 \end{aligned} \tag{13}$$

—

SCF of Other Fillet

Tension—Region 5 in Fig. 6*a*: $0.1 \leqq \rho/a \leqq 3.33\ (0.3 \leqq x \leqq 1.9)$ and $0.02 \leqq 2t/D \leqq 0.7$. The SCF of other fillets in tension can be expressed by Eq 14*a*. Figure 11*a* shows the value of Eq 14. The value of Eq 14*b* is shown in Fig. 9*a*.

$$\begin{aligned} K_t/K_{ts} & \\ = {} & \{1.0029 - 0.015487(\rho/a) + 0.023469(\rho/a)^2 \\ & - 0.01483(\rho/a)^3 + 0.0041529(\rho/a)^4 - 0.00042528(\rho/a)^5 \\ & + (0.0060789 + 0.43438(\rho/a) - 0.94447(\rho/a)^2 \\ & + 0.71668(\rho/a)^3 - 0.22567(\rho/a)^4 + 0.025260(\rho/a)^5)\lambda \\ & + (0.067368 - 2.0877(\rho/a) + 4.6167(\rho/a)^2 \\ & - 3.6814(\rho/a)^3 + 1.2153(\rho/a)^4 - 0.14154(\rho/a)^5)\lambda^2 \\ & + (-0.17844 + 2.0824(\rho/a) - 4.6746(\rho/a)^2 \\ & + 3.9009(\rho/a)^3 - 1.3342(\rho/a)^4 + 0.15964(\rho/a)^5)\lambda^3\} \\ & \times ([K_t/K_{ts}]_{\rho/a=1.25}) \end{aligned} \tag{14a}$$

$$\begin{aligned} [K_t/K_{ts}]_{\rho/a=1.25} = {} & 0.99991 - 0.54893\lambda - 0.97945\lambda^2 \\ & + 12.255\lambda^3 - 69.872\lambda^4 + 205.88\lambda^5 \\ & - 325.46\lambda^6 + 260.71\lambda^7 - 82.979\lambda^8 \end{aligned} \tag{14b}$$

Bending—Region 5 in Fig. 6*b*: $0.1 \leqq \rho/a \leqq 1.67\ (0.6 \leqq x \leqq 1.9)$ and $0.02 \leqq 2t/D \leqq 0.4$. The SCF of other fillets in bending can be expressed by Eq 15. Figure 11*a* shows the values of Eq 15*a*. The value of Eq 15*b* is shown in Fig. 9*b*:

$$\begin{aligned} K_t/K_{ts} = {} & \{0.99487 + 0.084386(\rho/a) - 0.54684(\rho/a)^2 \\ & + 1.5772(\rho/a)^3 - 2.1768(\rho/a)^4 + 1.3981(\rho/a)^5 \\ & - 0.33129(\rho/a)^6 + (-0.30923 + 0.48523(\rho/a) \\ & + 6.7080(\rho/a)^2 - 27.377(\rho/a)^3 + 41.194(\rho/a)^4 \\ & - 27.573(\rho/a)^5 + 6.7330(\rho/a)^6)\lambda + (1.2707 \end{aligned}$$

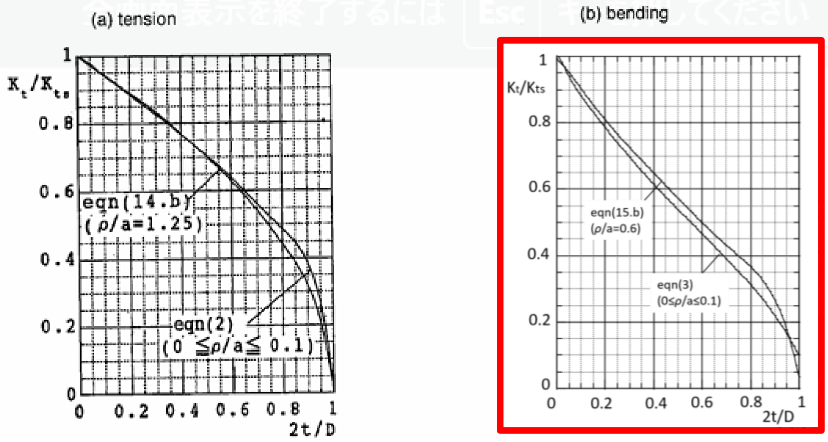


FIG. 7—K_t/K_{ts} *versus* $2t/D$. (a) *Tension.* (b) *Bending.*

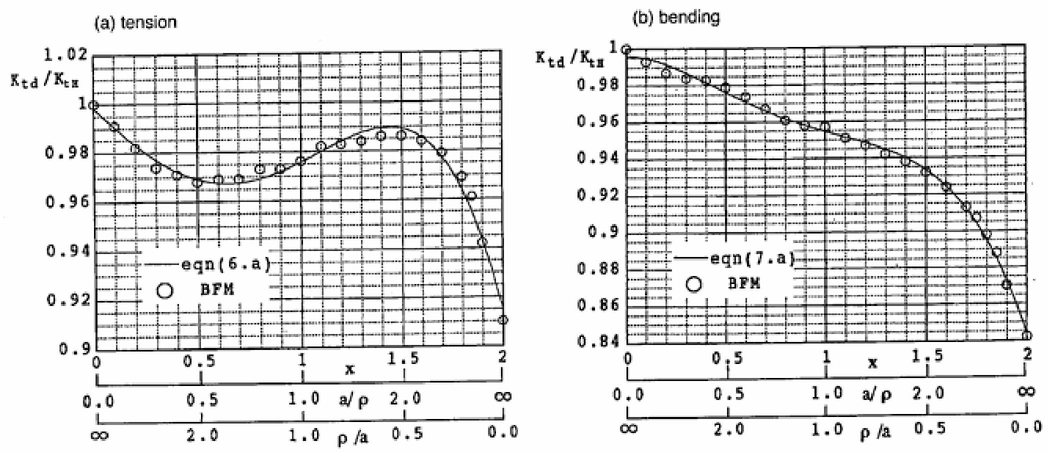


FIG. 8—K_{td}/K_{tH} *versus* a/ρ *or* ρ/a. (a) *Tension.* (b) *Bending.*

$$- 10.823(\rho/a) + 12.352(\rho/a)^2 + 42.601(\rho/a)^3$$
$$- 109.27(\rho/a)^4 + 88.725(\rho/a)^5 - 23.921(\rho/a)^6)\lambda^2$$
$$+ (-1.8099 + 20.357(\rho/a) - 47.047(\rho/a)^2$$
$$+ 5.7878(\rho/a)^3 + 83.638(\rho/a)^4$$
$$- 88.070(\rho/a)^5 + 26.131(\rho/a)^6)\lambda^3\}$$
$$\times ([K_t/K_{ts}]_{\rho/a=0.6}) \qquad (15a)$$

$$[K_t/K_{ts}]_{\frac{\rho}{a}=0.6} = 0.9945 - 0.7686x - 2.7356x^2 + 15.132x^3$$
$$-34.305x^4 + 35.826x^5 - 14.109x^6 \quad (15b)$$

As stated above, the set of formulas given by Eqs 1 to 15 give accurate stress concentration factors K_t for the whole range of fillets.

SCF of Any Dimensions of Fillet

By fitting smooth curves to Eqs 1–15, a convenient formula can be proposed. Here, to improve the accuracy, other approximate formulas are used. Also, the body force method is applied again to confirm accurate K_t values when some differences are observed between the equations. First, the modified Neuber formula K_{tN}^* for fillets is expressed by [7]

$$K_{tN}^* = \{(K_{ts} - 1)(K_{td} - 1)/((K_{ts}^* - 1)^m + (K_{ts} - 1)^m)^{1/m}\} + 1$$
$$Tension: m = 1.6, \quad Bending: m = 1.4 \qquad (16)$$

The error of equation (16) is estimated within about within 4 % for tension and 6 % for bending [8]. Then, the least-squares method is applied to K_t/K_{tN}^* where K_t is the value of Eqs 1–15. Figure 12 and Eq 17*a* and 17*b* express correction factors for K_t/K_{tN}^*. The

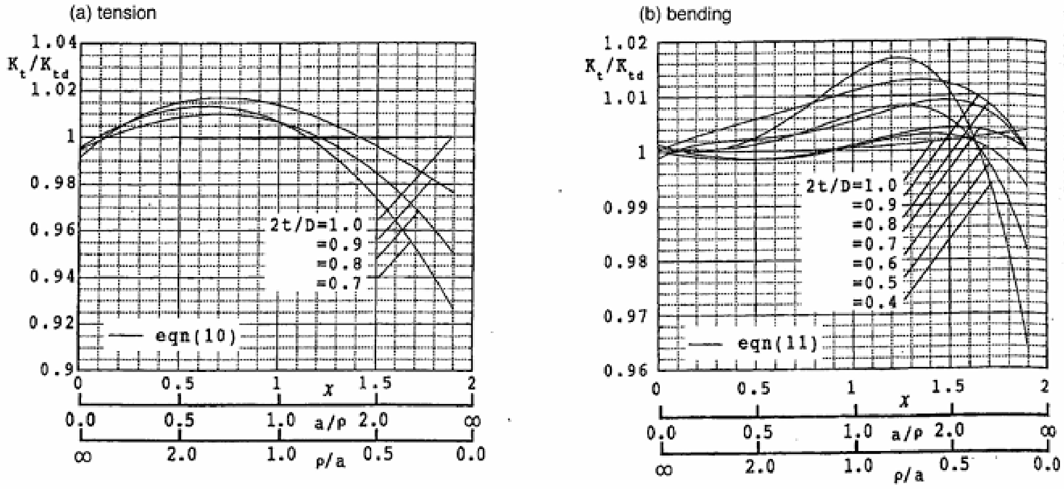


FIG. 9—K_t/K_{td} *versus* a/ρ *or* ρ/a. (a) *Tension.* (b) *Bending.*

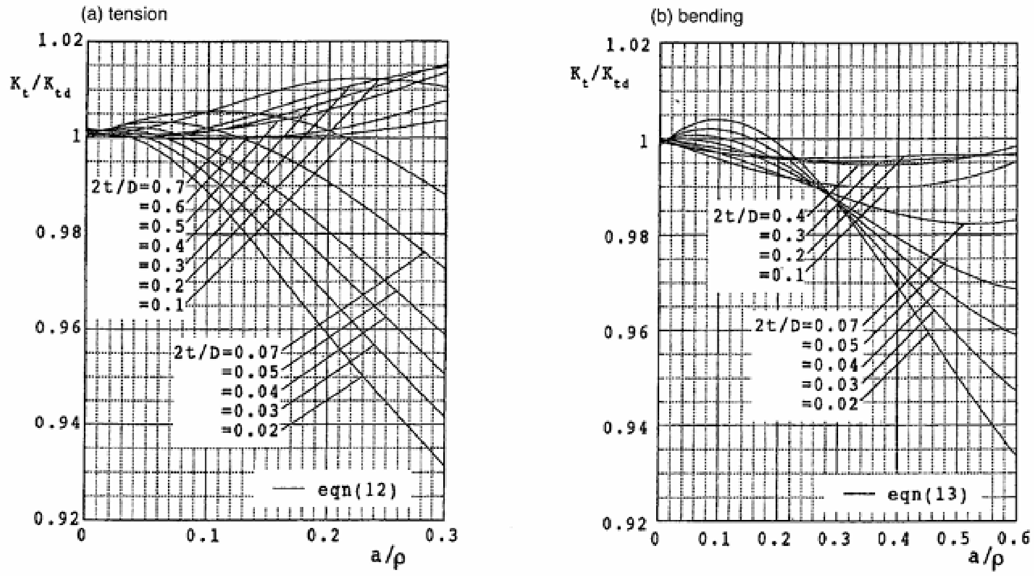


FIG. 10—K_t/K_{td} *versus* a/ρ *or* ρ/a. (a) *Tension.* (b) *Bending.*

formulas give SCFs within about 1 % error in most cases for any dimensions of fillet.

Tension $(0 \leq x \leq 2.0)$ $(0.07 \leq \lambda \leq 1.0)$

$$K_t/K_{tN}^* = (1.0001 + 0.0010812x - 0.00060452x^2) + (0.25406 - 0.27860x + 0.15750x^2)\lambda + (-1.2949 + 3.5075x - 1.6332x^2)\lambda^2 + (2.5378 - 9.4629x + 4.3688x^2)\lambda^3 + (-2.2499 + 9.9393x - 4.6004x^2)\lambda^4 + (0.75284 - 3.7040x + 1.7068x^2)\lambda^5 \quad (17a)$$

$(0 \leq \lambda \leq 0.07)$

$$K_t/K_{tN}^* = 1.0$$

Bending $(0 \leq \lambda \leq 1.0,\ 0 \leq x \leq 2.0)$

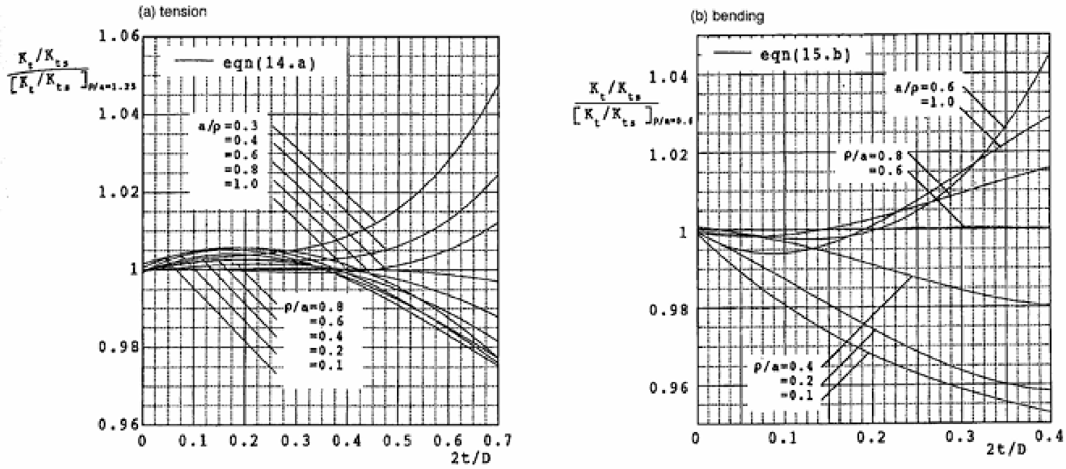


FIG. 11—$(K_t/K_{ts})/([K_t/K_{ts}]_{\rho/a=1.25})$, $(K_t/K_{ts})/([K_t/K_{ts}]_{\rho/a=0.6})$ *versus* $2t/D$. *(a) Tension. (b) Bending.*

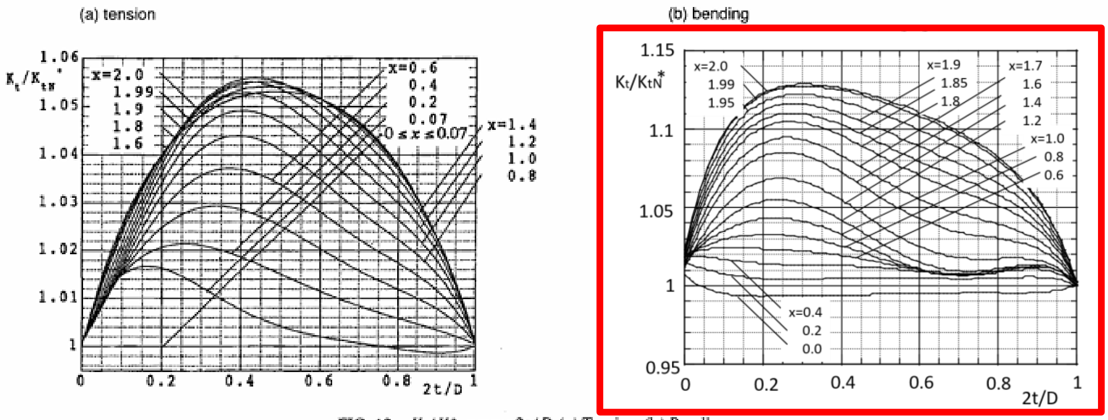


FIG. 12—K_t/K_{tN}^* *versus* $2t/D$ *(a) Tension. (b) Bending.*

$$
\begin{aligned}
K_t/K_{tN}^* &= (1.0067 + 0.0518x - 0.0631x^2 + 0.0207x^3) \\
&+(-0.2108 + 0.7798x - 0.5783x^2 + 0.2616x^3)\lambda \\
&+(1.2819 - 8.4349x + 10.292x^2 - 3.81x^3)\lambda^2 \\
&+(-3.8272 + 34.309x - 46.515x^2 + 16.592x^3)\lambda^3 \\
&+(6.0506 - 64.639x + 87.726x^2 - 30.427x^3)\lambda^4 \\
&+(-4.8293 + 57.266x - 75.322x^2 + 25.406x^3)\lambda^5 \\
&+(1.5283 - 19.334x + 24.462x^2 - 8.0434x^3)\lambda^6 \quad (17b)
\end{aligned}
$$

Figure 13 shows SCF charts obtained from these equations.

Conclusions

In this paper, stress concentration formulas K_t of a flat bar with fillets under tension and bending are considered on the basis of the exact solutions now available for the limiting cases together with accurate numerical results obtained by the body force method. The conclusions can be summarized as follows:

1. For the limiting cases of deep and shallow fillets, the body force method was used to calculate the stress concentration factors (SCFs). Then, the formulas for deep (d) and shallow (s) fillets were obtained as K_{td} and K_{ts} values.
2. On the one hand, upon comparison of K_t and K_{td}, it was found that K_t is nearly equal to K_{td} if the fillet is deep or blunt.
3. On the other hand, if the fillet is sharp or shallow, K_t is controlled mainly by K_{ts} and the fillet depth.

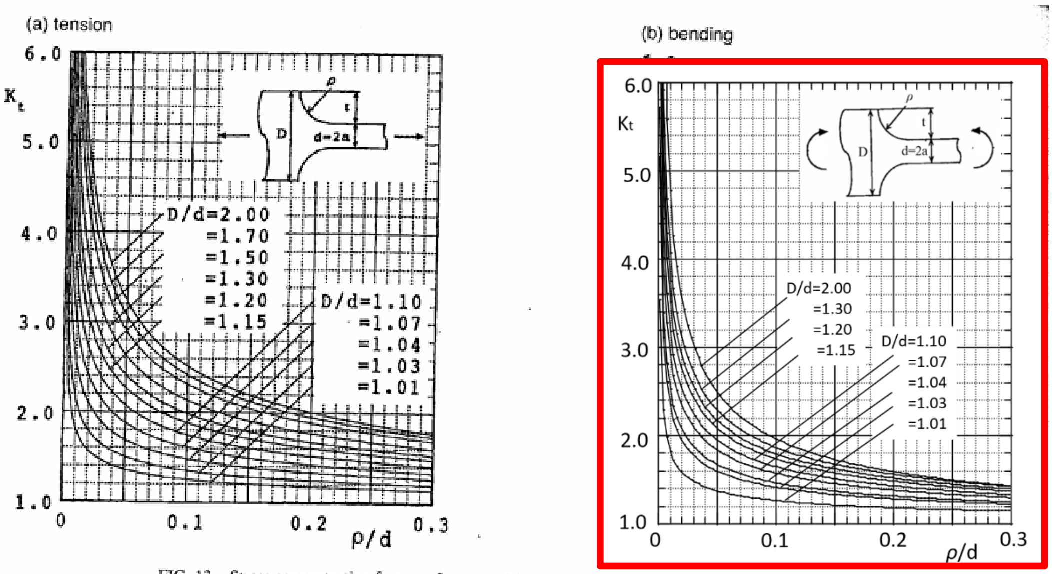


FIG. 13—*Stress concentration factors of a stepped flat bar with shoulder fillets: (a) Tension. (b) Bending.*

4. The fillet shape can be classified into several groups according to the fillet radius and fillet depth. The least-squares method can be applied for the calculation of K_t/K_{td} and K_t/K_{ts}.
5. Finally, convenient formulas are proposed that are useful for any dimensions of fillet in a flat bar. The formulas give SCFs with less than about 1 % error in most cases for any dimensions of fillet under tension and bending.

Acknowledgments

The authors wish to express their thanks to the members of their group, especially Mr. K. Matsunaga and Mr. Y. Eto, who carried out much of the constructional work.

References

[1] Hasebe, N. and Iida, J., "Notch Mechanics for Plane and Thin Plate Bending Problems," *Engineering Fracture Mechanics*, Vol. 37, 1990, pp. 87–99.

[2] Hasebe, N., "Stress Analysis of a Semi-Infinite Plate with a Triangular Notch or Mound," *Proceedings of the Japan Society of Civil Engineers*, Vol. 194, 1971, pp. 29–40.

[3] Noda, N.-A. and Takase, Y., "Stress Concentration Formulas Useful for All Notch Shapes in a Flat Test Specimen Under Tension and Bending," *Journal of Testing and Evaluation*, Vol. 30, No. 5, 2002, pp. 369–381.

[4] Noda, N. A., Yamasaki, T., Matsuo, K., and Takase, Y., "Interaction Between Fillet and Crack in Round and Flat Test Specimens," *Engineering Fracture Mechanics*, Vol. 50, 1995, pp. 385–405.

[5] Noda, N.-A. and Takase, Y., "Study on Stress Concentration Factor of Fillet in Stepped Round and Flat Bars Under Tension and Bending," *Transactions, Japan Society of Mechanical Engineers*, Vol. 62, 1996, pp. 787–793 (in Japanese).

[6] Peterson, R. E., *Stress Concentration Factors*, Wiley, New York, 1974.

[7] Neuber, H., *Kerbspannungslehre*, Spring-Verlag, Berlin, 1957.

[8] Noda, N.-A., Takase, Y., and Monda, K., "Stress Concentration Factors for Shoulder Fillets in Round and Flat Bars under Various Loads," *International Journal of Fatigue*, Vol. 19, 1997, pp. 75–84.

[9] Noda, N.-A. and Takase, Y., "Stress Concentration Formulas Useful for Any Shape of Notch in a Round Test Specimen Under Tension and Bending," *Fatigue and Fracture of Engineering Materials and Structures*, Vol. 22, 1999, pp. 1071–1082.